Proving termination through conditional termination

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supported by MINECO/FEDER project TIN2015-69175-C4-3-R

Bergen, Norway
October 2016

Overview of the talk

1 Introduction

2 SMT/Max-SMT solving

3 (Conditional) Invariant generation

4 Termination analysis

5 Compositional termination analysis

6 Conclusions and current work

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SMT solving

We make extensive use of SMT solvers inside our program analysis tools.

Input: Given a boolean formula φ over some theory T.

Question: Is there any model that satisfies the formula?

Example: $T =$ non-linear (polynomial) integer/real arithmetic.

$$(x^2 + y^2 > 2 \;\vee\; x \cdot z \leq y \;\vee\; y \cdot z < z^2) \;\wedge\; (x > y \;\vee\; 0 < z)$$

$$\{x = 0,\; y = 1,\; z = 1\}$$

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- Reals: decidable (Tarski) **but** algorithms have prohibitive complexity.

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- Need to handle large formulas with non-linear arithmetic and complex boolean structure.
- Barcelogic has shown to be the best SMT-solver proving satisfiability of this kind of problems.

Optimization problems

(Weighted) Max-SMT problem

Input: Given an SMT formula $\varphi = C_1 \wedge \ldots \wedge C_m$ in CNF, where some of the clauses are hard and the others soft with a weight.

Output: An assignment for the hard clauses that minimizes the sum of the weights of the falsified soft clauses.

$$(x^2 + y^2 > 2 \ \vee \ x \cdot z \leq y \ \vee \ y \cdot z < z^2) \ \wedge \ (x > y \ \vee \ 0 < z \ \vee \ w(5)) \wedge \ldots$$

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We focus on inductive invariants.

Constraint-based invariant generation

We inspire ourselves with the constraint-based method [CSS'03].

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Keys:

- Use a template for candidate invariants.

$$c_1 x_1 + \ldots + c_n x_n + d \leq 0$$

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- Transform it using Farkas' Lemma into an $\exists$ problem over non-linear arithmetic.

Scalar invariant generation: Example

Square root of a natural number N:

```
int isqrt(int N) {
  int a = 0, s = 1, t = 1;
  // Inv:  c1 a + c2 s + c3 t + d ≤ 0
  while (s ≤ N) {
    a = a + 1;
    s = s + t + 2;
    t = t + 2;
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  return a;
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$\exists\ c_1,\ c_2,\ c_3,\ d\quad \forall\ a,\ s,\ t$

$\underbrace{true \Longrightarrow c_1 \cdot 0 + c_2 \cdot 1 + c_3 \cdot 1 + d \leq 0} \ \wedge$ Initiation condition

$$\underbrace{s \leq N \wedge c_1 \cdot a + c_2 \cdot s + c_3 \cdot t + d \leq 0 \Longrightarrow c_1 \cdot (a+1) + c_2 \cdot (s+t+2) + c_3 \cdot (t+2) + d \leq 0}_{\text{consecution condition}}$$

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$\exists\ c_1,\ c_2,\ c_3,\ d\ \ \forall\ a,\ s,\ t$

$\underbrace{c_2 + c_3 + d \leq 0}\ \wedge$ Initiation condition

$$\underbrace{s \leq N \wedge c_1 \cdot a + c_2 \cdot s + c_3 \cdot t + d \leq 0 \Longrightarrow c_1 \cdot a + c_2 \cdot s + (c_2 + c_3) \cdot t + c_1 + 2c_2 + 2c_3 + d \leq 0}_{\text{consecution condition}}$$

▸ Return

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Apply Farkas' Lemma to remove $\forall$ a, s, t

Use Barcelogic to solve the non-linear SMT problem!

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$$\{c_1 = -2, c_2 = 0, c_3 = 1, d = -1\}$$

Scalar invariant generation: Example

Square root of a natural number N:

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int isqrt(int N) {
  int a = 0, s = 1, t = 1;
  // Inv:  −2a + 0s + 1t − 1 ≤ 0
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A formula is a conditional (inductive) invariant at a program location if:

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Key: We prefer invariants but we can live with conditional invariants

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Encode the problem using Max-SMT,

We use Barcelogic to solve it.

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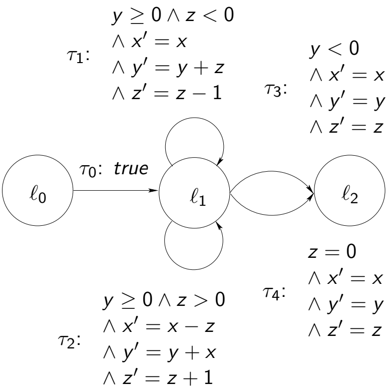
Running example

```
int main() {
  int x, y, z;
  x = nondet();
  y = nondet();
  z = nondet();
  while (y ≥ 0 && z ≠ 0) {
    if (z < 0) { y = y + z;
                 z = z − 1;
    } else { x = x − z;
             y = y + x;
             z = z + 1;
    }
  }
}
```



Ranking functions and Conditional Invariants

In order to discard a transition τ_i we need to find a ranking function f over the integers such that:

1. $\tau_i \Longrightarrow f(x_1, \ldots, x_n) \geq 0$ (bounded)
2. $\tau_i \Longrightarrow f(x_1, \ldots, x_n) > f(x'_1, \ldots, x'_n)$ (strict-decreasing)
3. $\tau_j \Longrightarrow f(x_1, \ldots, x_n) \geq f(x'_1, \ldots, x'_n)$ for all j (non-increasing)

Use a linear template for the ranking function as well.

Ranking functions and Invariants: Combined problem

In order to prove properties of the ranking function we may need to generate invariants.

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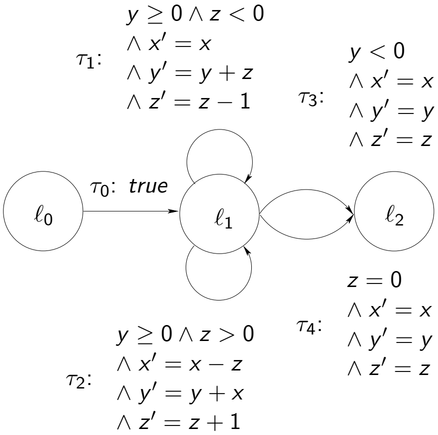
Considering conditional invariants give more chances to the solver

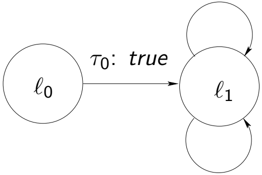
But we get a conditional termination proof

Running example



Running example

$$\tau_1: \quad \begin{array}{l} y \geq 0 \wedge z < 0 \\ \wedge\ x' = x \\ \wedge\ y' = y + z \\ \wedge\ z' = z - 1 \end{array}$$



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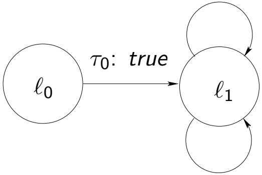

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- $z < 0$ is a conditional invariant at location ℓ_1
- y is a ranking function
  1. τ_1 is bounded and strictly decreasing
  2. τ_2 is disabled

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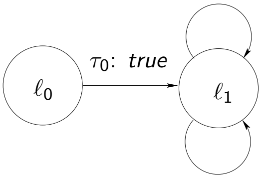

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We have a conditional proof:

The system terminates if the condition $z < 0$ holds at l_0 (or τ_0)

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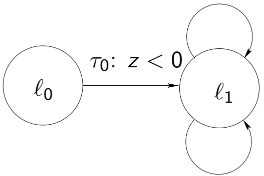

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Running example: Narrowing

In order to complete the termination proof we have to consider the complementary problem.

Narrow the transitions removing all states that we already now that are terminating.

We can do better than just add the negation of the condition in the entry.

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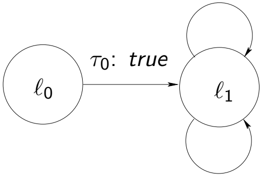

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We know more!:

whenever $z < 0$ holds at ℓ_1 the system terminates

Skip

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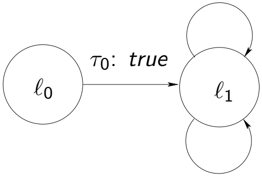

$$\tau_2: \quad \begin{array}{l} y \geq 0 \wedge z > 0 \\ \wedge\ x' = x - z \\ \wedge\ y' = y + x \\ \wedge\ z' = z + 1 \end{array}$$

Narrow the transition system according to this:

whenever $z < 0$ holds at ℓ_1 the system terminates

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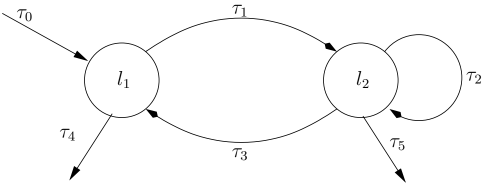
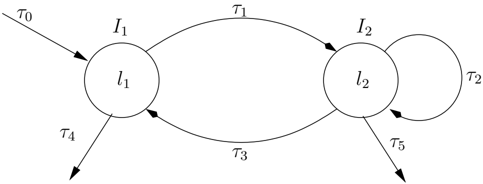
Narrowing

Assume we have the following transition system:



Narrowing

After sending the problem to our Max-SMT solver we get:



- Conditional invariant I_1 at location l_1.
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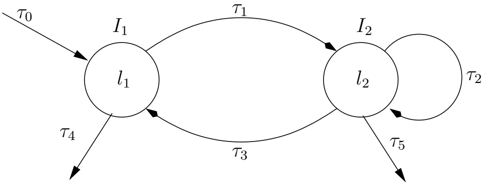

- Conditional invariant I_1 at location l_1.
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- If I_1 holds in location l_1 then I_2 holds in location l_2.
- I_2 is preserved in l_2.
- If I_2 holds in location l_2 then I_1 holds in location l_1.
- If I_2 holds in l_2 and I_2 holds in l_2 then it terminates.

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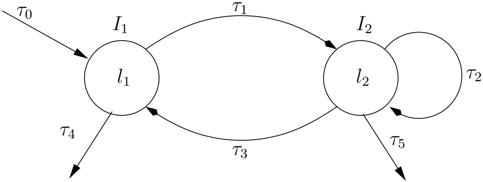

- Conditional invariant I_1 at location l_1.
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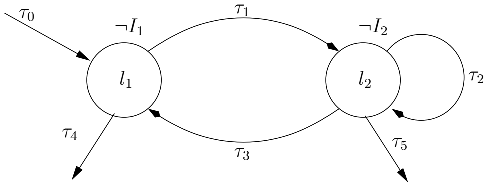
Therefore

- If I_1 holds in location l_1 we are done.
- If I_2 holds in location l_2 we are done.

Narrowing

After narrowing

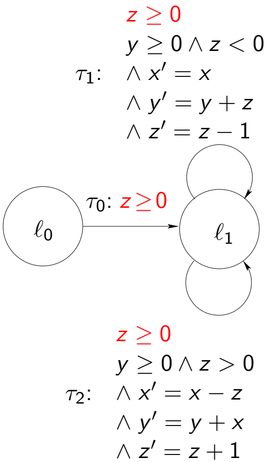


Remains to be proved

Therefore

- The entry τ_0 is narrowed with $\neg I_1'$
- Transition τ_1 is narrowed with $\neg I_1$ and $\neg I_2'$
- Transition τ_2 is narrowed with $\neg I_2$ and $\neg I_2'$
- Transition τ_3 is narrowed with $\neg I_2$ and $\neg I_1'$

Running example: Narrowing

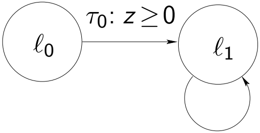


Narrow the transition system according to this:

whenever $z < 0$ holds at ℓ_1 the system terminates

Running example. Narrowing

After simplifying the transition system we get:



$$\tau_2: \quad \begin{aligned} & y \geq 0 \wedge z > 0 \\ & \wedge\, x' = x - z \\ & \wedge\, y' = y + x \\ & \wedge\, z' = z + 1 \end{aligned}$$

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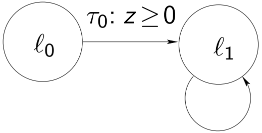


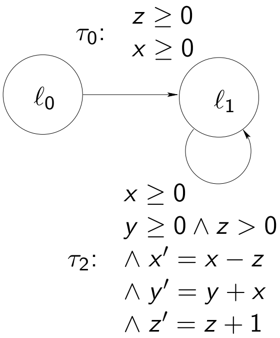
$$\tau_2: \quad \begin{array}{l} y \geq 0 \wedge z > 0 \\ \wedge\ x' = x - z \\ \wedge\ y' = y + x \\ \wedge\ z' = z + 1 \end{array}$$

Conditionally terminates:

- $x < 0$ is a conditional invariant at location ℓ_1
- y is a ranking function
  1. τ_2 is bounded and strictly decreasing

Running example. Narrowing

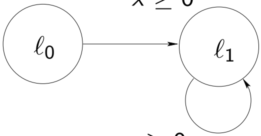
Narrowing again with the complement of $x < 0$ we get:



Running example. Narrowing

Narrowing again with the complement of $x < 0$ we get:

τ_0: $z \geq 0$, $x \geq 0$



$$\tau_2: \begin{array}{l} x \geq 0 \\ y \geq 0 \wedge z > 0 \\ \wedge\, x' = x - z \\ \wedge\, y' = y + x \\ \wedge\, z' = z + 1 \end{array}$$

Which terminates with x as a ranking function

Advantages of conditional termination

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proving termination by cases

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but if this is not possible (in a given time limit)

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but if this is not possible (in a given time limit)

it can provide a conditional proof (soft constraints) which give us a progress.

An additional advantage (key in some case):

If we cannot prove termination of the narrowed transition system

we can use it to try to prove non-termination

as the non-terminating execution (if any) should be there!

Overview of the talk

1 Introduction

2 SMT/Max-SMT solving

3 (Conditional) Invariant generation

4 Termination analysis

5 Compositional termination analysis

6 Conclusions and current work

Scalable Termination Analysis

Aim: prove termination in large programs (several consecutive loops).

New approach:

1. Obtain a conditional termination proof.
2. Check (compositionally) the condition as a Safety property.

Simple example:

```
assume(x > y && y ≥ 0);
while (y > 0) {
  x = x - 1;
  y = y - 1;
}
while (y < 0) {
  y = y + x;
}
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1. Obtain a conditional termination proof.
2. Check (compositionally) the condition as a Safety property.

Simple example:

```
assume(x > y && y ≥ 0);
while (y > 0) {
  x = x - 1;
  y = y - 1;
}
assert(x > 0); Rank: -y
while (y < 0) {
  y = y + x;
}
```

Compositional analysis through Conditional Termination

Aim: verify termination in large programs (several consecutive loops).

Key ideas:

- Generate conditional proofs:
  - Find conditional invariants implying termination
- Check the condition as a Safety property of previous loops.

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Key ideas:

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- Check the condition as a Safety property of previous loops.
- In case of failure of the Safety checker
  Narrow the loop and try again!

We can handle every loop (or SCC in general) independently

Experiments

Our techniques have been implemented in VeryMax(already presented)

These techniques can be highly parallelized (sharing few information).

Compared to last year competitors in TermComp on (335) Integer C programs

Tool	Terminating
AProVE	208(5)
HipTNT+	210(5)
UltimateBuchiAutomizer	207
VeryMax	213

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Conclusions

Two main conclusions:

- Using SMT and Max-SMT, automatic generation of conditional invariants and ranking function becomes feasible.

- In constraint-based program analysis it is often better to consider that we have optimization problems rather than satisfiability problems!

Under development:

- Combine conditional termination and non-termination analysis.
- Use conditional termination to provide witness of termination. For instance, it has applications to check reachability.

Future developments?:

- Generate linear upper bounds

Thank you!